Problem 51

Consider the physical quantities s, v, a, and t with dimensions $[s] = \text{L}$, $[v] = \text{LT}^{-1}$, $[a] = \text{LT}^{-2}$, and $[t] = \text{T}$. Determine whether each of the following equations is dimensionally consistent. (a) $v^2 = 2as$; (b) $s = vt^2 + 0.5at^2$; (c) $v = s/t$; (d) $a = v/t$.

Solution

Part (a)

Check the units of both sides.

$$[v^2] \stackrel{?}{=} [2as]$$

$$[v]^2 \stackrel{?}{=} [2][a][s]$$

$$(\text{LT}^{-1})^2 \stackrel{?}{=} 1 \cdot \text{LT}^{-2} \cdot \text{L}$$

$$\text{L}^2\text{T}^{-2} = \text{L}^2\text{T}^{-2}$$

Both sides have the same dimensions, so this equation is dimensionally consistent.

Part (b)

Check the units of both sides.

$$[s] \stackrel{?}{=} [vt^2 + 0.5at^2]$$

$$\text{L} \stackrel{?}{=} [vt^2] + [0.5at^2]$$

$$\stackrel{?}{=} [v][t^2] + [0.5][a][t^2]$$

$$\stackrel{?}{=} (\text{LT}^{-1}) \cdot \text{T}^2 + 1 \cdot \text{LT}^{-2} \cdot \text{T}^2$$

$$\neq \text{LT} + \text{L}$$

Both sides have different dimensions, so this equation is not dimensionally consistent. vt^2 should be changed to vt to make it consistent.

Part (c)

Check the units of both sides.

$$\begin{aligned}
[v] &\stackrel{?}{=} \left[\frac{s}{t}\right] \\
\mathrm{LT}^{-1} &\stackrel{?}{=} \frac{[s]}{[t]} \\
&\stackrel{?}{=} \frac{\mathrm{L}}{\mathrm{T}} \\
&= \mathrm{LT}^{-1}
\end{aligned}$$

Both sides have the same dimensions, so this equation is dimensionally consistent.

Part (d)

Check the units of both sides.

$$\begin{aligned}
[a] &\stackrel{?}{=} \left[\frac{v}{t}\right] \\
\mathrm{LT}^{-2} &\stackrel{?}{=} \frac{[v]}{[t]} \\
&\stackrel{?}{=} \frac{\mathrm{LT}^{-1}}{\mathrm{T}} \\
&= \mathrm{LT}^{-2}
\end{aligned}$$

Both sides have the same dimensions, so this equation is dimensionally consistent.